'ROTATIONAL DYNAMICS'

|1| → Moment of Inertia (I) → In a Angular Motion property of oppose to change the angular velocity.

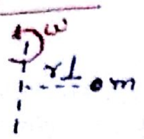
* unit → $kg m \times m^2$ (S.I).

Mathmatically, M.O.I is equal to Mass into square of ⊥ distance from axis of Rotation.

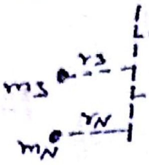
Imp * M.O.I is Tensor quantity → Not fallow vector Addition.

|A| → M.O.I of point Mass →

$\boxed{I = mr^2}$ *



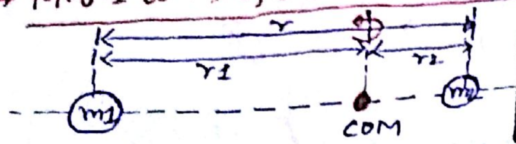
|B| → MOI of 'N' Mass System



* $\boxed{I = m_1 r_1^2 + m_2 r_2^2 + \dots m_N r_N^2}$

$\boxed{I = \sum_{i=1}^{i=N} m_i r_i^2}$ *

|C| → M.O.I w.r.t of C.O.M of 2 Mass System



$$r_1 = \frac{m_2 r}{m_1 + m_2}$$

$$r_2 = \frac{m_1 r}{m_1 + m_2}$$

$$I_{C.M} = m_1 r_1^2 + m_2 r_2^2$$

$$I_{C.M} = \left(\frac{m_1 m_2}{m_1 + m_2}\right) r^2$$

$$\boxed{I_{CM} = \mu r^2}$$

* μ = Reduce Mass of System.

Standard

* For two Mass System

$$\mu = \frac{m_1 m_2}{m_1 + m_2} \Rightarrow \frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

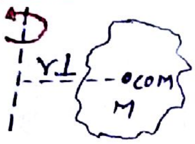
* For Three Mass System.

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} + \frac{1}{m_3}$$

* For 'N' Mass System

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2} + \dots + \frac{1}{m_N}$$

|D| → M.O.I of continious Mass distribution



* $\boxed{I = Mr_\perp^2}$ *

$$\boxed{\int dI = \int dMr^2}$$

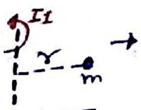
 # M.O.I depent on

* Mass of system
* Mass distribution
* Axis of Rotation.
* position of Axis of Rotation.

 # M.O.I Not depend on.

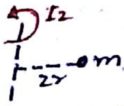
* Angular disp.
* Angular velocity.
* Angular Accl^n.
* Torque
* || dimension of Axis of Rotation.

EX → |i| →



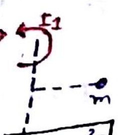
$\boxed{I_1 = mr^2}$

→

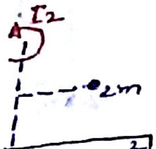


$I_2 = m(2r)^2 = \boxed{4mr^2}$

|ii| →



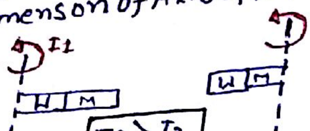
$\boxed{I_1 = mr^2}$



$\boxed{I_2 = 2mr^2}$

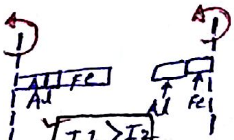
$\boxed{I_2 > I_1}$

|iii| →



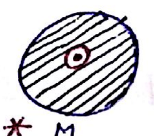
$\boxed{I_1 > I_2}$

|iv| →



$\boxed{I_1 > I_2}$

|v| → ** Normal cycle — Racing cycle



M R I_1 — M R I_2

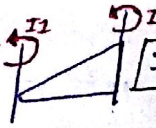
* $\boxed{I_1 > I_2}$

H.R ***
|vi| → Liq or Raw Egg M.O.I > Boiled Egg of MOI. 2016 JIPMER

$\boxed{I_1 > I_2}$ *

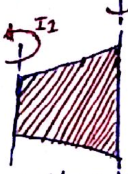
NOTE → When Earth pole Ice is melt, than M.O.I ↑, ω ↓, Length of Day ↑.

|vii| →



$\boxed{I_2 > I_1}$

|viii| →



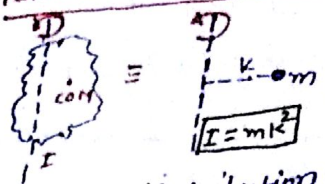
$\boxed{I_1 > I_2}$

NOTE → If External Force of system is zero than position of centre of mass remain unchanged in this condition If anyone mass system movees than another mass will also to maintain position of com. In this condition mass of the system is called Reduced Mass.

Standard

⊥ Distance of point (x, y, z) From
* x-axis $= \sqrt{y^2+z^2}$
* y-axis $= \sqrt{x^2+z^2}$
* z-axis $= \sqrt{x^2+y^2}$

Radius of Gyration (K)



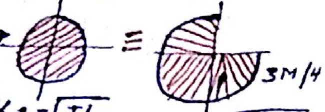
$\boxed{K = \sqrt{\frac{I}{m}}}$ *

$\boxed{I = mK^2}$

NOTE → * Radius of gyration depend on Mass distribution & Axis of Rotation. It is Independent from mass of object.

* In asymmetrical attachment or, detachment radius of gyration Remain unchange.

Ex →

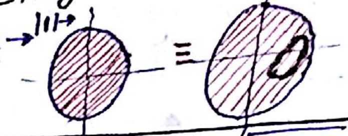


$K_1 = \sqrt{I/M}$ $\quad K_2 = \sqrt{\frac{\frac{3}{4}I}{3/4M}}$

$\boxed{K_2 = \sqrt{I/M} = K_1}$

In case of insymmetrical attachment or, detachment radius of gyration is change.

Ex → |i| →



$\boxed{K_{net} = \sqrt{\frac{I_{net}}{M_{net}}} = \sqrt{\frac{I_1+I_2}{M_1+M_2}}}$

|ii| → I_1

$\boxed{K = \sqrt{\frac{I_1-I_2}{M_1-M_2}}}$

For 'n' point mass system

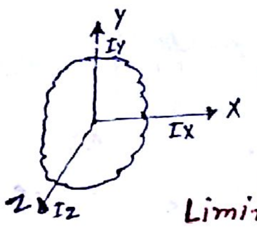
$\boxed{K = \sqrt{\frac{I_{net}}{m_{net}}} = \sqrt{\frac{I_1+I_2+\cdots I_N}{m_1+m_2+\cdots m_N}}}$

$\boxed{K = \sqrt{\frac{m_1r_1^2+m_2r_2^2+\cdots m_nr_N^2}{m_1+m_2+\cdots m_n}}}$

Axis theorum:

|a| → ⊥ axis theorum → * valid only for 2-D object.

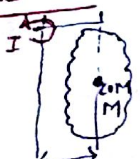
* Same of MOI of two ⊥ axis wich is ⊕nt in a plane of 2-D object is equal to ⊥ axis wich is pass from Intercept point of axis.



* If obj. ⊕nt in a 'x-y' plane ⇒ $I_x + I_y \Rightarrow I_z$
* If obj. ⊕nt in a 'y-z' plane ⇒ $I_y + I_z \Rightarrow I_x$
* If obj ⊕nt in a 'z-x' plane ⇒ $I_y = I_z + I_x$

Limitation → ⊥ axis theorum applied only on two dimension object.

|b| → || axis theorum →

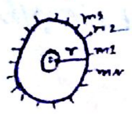


$\boxed{I = I_{c.m} + Md^2}$

M.O.I Rigid body (continous mass distribution)

|1| → RING

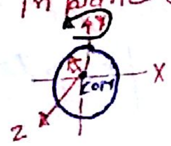
|a| → Axis which is passed from COM & ⊥ to the plane (Geomertical Axis)



$\int dI = \int dmr^2$

$\boxed{I = Mr^2}$ *

|b| → Axis which is passed from COM & ⊕nt in plane (Diametric axis)



$I_x = I_y \quad I_z = MR^2$

$I_z = I_x + I_y$

$2I_x = I_z$ $\boxed{I_x = \frac{I_z}{2} = \frac{Mr^2}{2} = I_y}$ *

|c| Tangential Axis

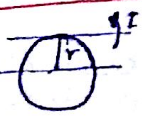
|i| → ⊥ to plane

$I = I_{cm} + MR^2$

$Mr^2 + Mr^2$

$2Mr^2$

|ii| → || to the plane



$I = I_{cM} + MR^2$

$\frac{Mr^2}{2} + Mr^2$

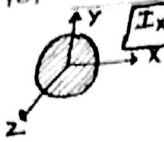
$= \frac{3}{2}MR^2$

|2|→ DISK

|a|→ Axis passed from COM & ⊥ to plane

$$\boxed{I = \frac{MR^2}{2}}$$ *

|b|→ Dimetric Axis



$$\boxed{I_x = \frac{I_2}{2} = \frac{Mr^2}{4} = I_y}$$ *

|c|→ |i|→ ⊥ to plane

$$\boxed{\frac{3}{2}MR^2}$$

|ii|→ ∥ to the plane

$$\boxed{\frac{5}{4}MR^2}$$

|3|→ Solid Sphere

|i|→ Diametric axis / passed from COM.

$$\boxed{I = \frac{2}{5}MR^2}$$ *

|ii|→ Tangential Axis

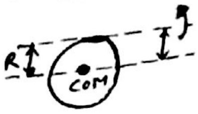
$$I = I_{cm} + MR^2$$
$$= \frac{2}{5}MR^2 + MR^2 = \boxed{\frac{7}{5}MR^2}$$

|4|→ Hollow Sphere

|i|→ M.O.I of Geometric Axis / Dimetric axis / Pas from COM.

$$\boxed{I = \frac{2}{3}MR^2}$$

|ii|→ Tangential Axis

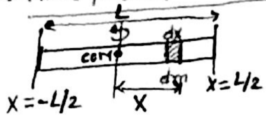


$$I = I_{cm} + MR^2$$
$$= \frac{2}{3}MR^2 + MR^2 = \boxed{\frac{5}{3}MR^2}$$

NOTE → If In Q. axis is not define than take Geometric Axis.

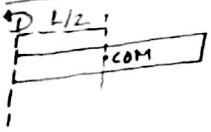
|5|→ ROD

|i|→ Axis passed from COM & ⊥ to the length



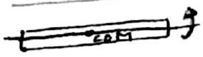
$$\int dI = \int dm\, x^2$$
$$= \int_{-L/2}^{L/2} \left(\frac{M}{L}dx\right)x^2 \Rightarrow \boxed{I = \frac{ML^2}{12}}$$ *

|ii|→ Tangential Axis, ⊥ to the length



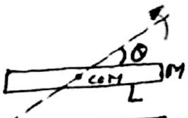
$$I = \frac{ML^2}{12} + M(L/2)^2$$
$$\boxed{I = \frac{ML^2}{3}}$$

|iii|→ Axis Along the Length



$$\boxed{I = 0}$$ **

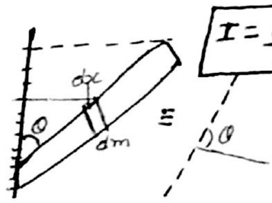
|iv|→



$$\boxed{I = \frac{ML^2}{12}\sin\theta}$$

[Angle from surface Not from Normal]

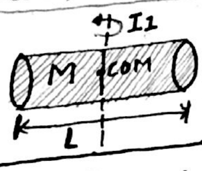
|v|→



$$\boxed{I = \frac{ML^2}{3}\sin^2\theta}$$

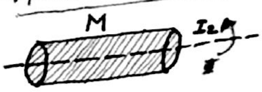
|6|→ Solid cylinder

|i|→ Passed from COM & ⊥ to the length



$$\boxed{I_1 = \frac{ML^2}{12} + \frac{MR^2}{4}}$$

|ii|→ passed from COM & ∥ to the length

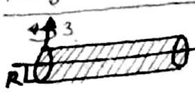


$$\boxed{I_2 = \frac{MR^2}{2}}$$

Standard *

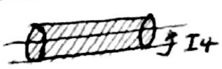
$$\boxed{I_1 = I_2} \Rightarrow \frac{ML^2}{12} + \frac{MR^2}{4} = \frac{MR^2}{2} \Big/ \frac{ML^2}{12} = \frac{MR^2}{4} \Rightarrow \boxed{\frac{L}{R} = \sqrt{3} : 1}$$ * $$\boxed{L = \sqrt{3} : 1}$$

|iii|→ Tangential Axis ⊥ to the length



$$\boxed{I_3 = \frac{ML^2}{3} + \frac{MR^2}{4}}$$

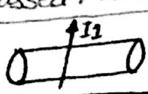
|iv|→ Tangential axis ∥ to the Length



$$\boxed{I_4 = \frac{3}{2}MR^2}$$

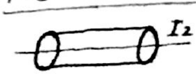
|7|→ Hollow cylinder

|i|→ Passed From COM & ⊥ to the Length



$$\boxed{I_1 = \frac{ML^2}{12} + \frac{MR^2}{2}}$$

|ii|→ Passed From COM & ∥ to Length



$$\boxed{I_2 = MR^2}$$ *

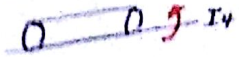
Standard *

$$\boxed{I_1 = I_2} \Rightarrow \frac{ML^2}{12} + \frac{ML^2}{2} = MR^2, \quad \frac{ML^2}{12} = \frac{MR^2}{2} \quad \boxed{\frac{L}{R} = \sqrt{6} : 1}; \boxed{L = \sqrt{6}R}$$

|iii| → Tangential axis ⊥ to the length

$\boxed{I_3 = \frac{ML^2}{3} + \frac{MR^2}{2}}$

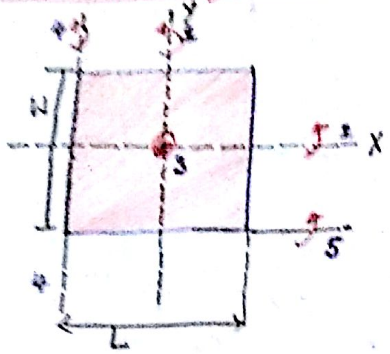
|iv| → Tangential axis ∥ to the length (Generator Axis)*



$\boxed{I_4 = 2MR^2}$

↓ starter of generator ent along the length

|8| → M.O.I Rectangular plate



*|i| → $I_1 = \frac{ML^2}{12}$

*|ii| → $I_2 = \frac{MW^2}{12}$

*|iii| → ⊥ Axis theorum

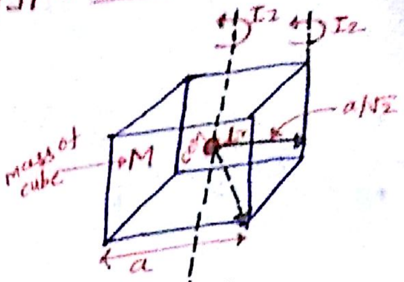
$I_3 = I_1 + I_2$

$I_3 = \frac{ML^2}{12} + \frac{MW^2}{12}$

*|iv| → $I_4 = \frac{ML^2}{3}$

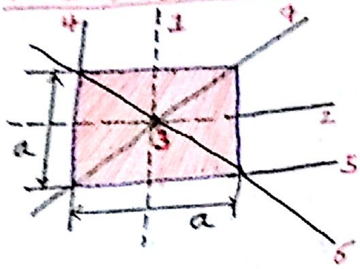
*|v| → $I_5 = \frac{MW^2}{3}$

|9| → M.O.I. of cube



$\boxed{I_1 = \frac{Ma^2}{6} \quad I_2 = \frac{2}{3}Ma^2}$

|10| → M.O.I of square plate



|i, ii| → $I_1 = \frac{Ma^2}{12} = I_2$

|iii| → $I_3 = I_1 + I_2 = \frac{Ma^2}{6}$

|iv, v| → $I_4 = I_5 = \frac{Ma^2}{3}$

#

$I_3 = I_6 + I_7 \; (I_6 = I_7)$

$2I_6 = I_3$

$I_6 = I_7 = \frac{I_3}{2} = \boxed{\frac{Ma^2}{12}}$

|11| → some part is Removed from object than M.O.I. of Remaning part.

$\boxed{I_{\text{Remaning part}} = I_{\text{complete object}} - I_{\text{Removed part}}}$

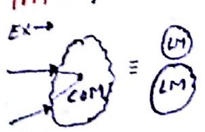
* w.r.t same axis of Rotation.

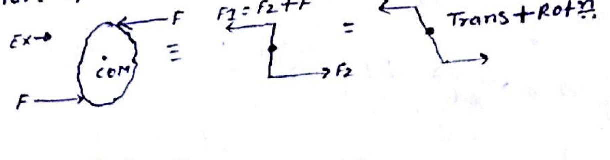
TORQUE

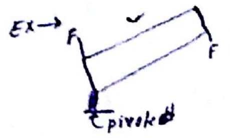
Turning Effect of Rotation is called Torque. & due to torque body rotate w.r.t Respective axis & It is possible When –

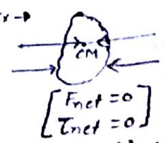
*|i| → couple of Force is Generate on same body in opposide direction.

*|ii| → & Line of Action of Force is not pass From C.O.M.

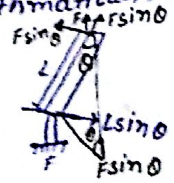








|i| # Mathmatically, Torque is product of Force & ⊥r distance from axis of Rotation.



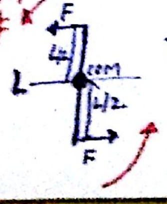
* $\boxed{\tau = F(\sin\theta)L \quad \tau = F(L\sin\theta) \quad \tau = F(r\perp)}$

$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$

NOTE → Angular disp, Angular velo, Angular Accln, Torque & Angular Momentum is Axial vector & Its direction Find from Right hand thumb Rule.

* sign → [* If A.C.W → ⊕ve (τ) ; * If C.W → ⊖ve (τ)]

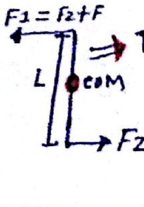
|ii| #



$\tau = F\left(\frac{L}{2}\right) + F\left(\frac{L}{2}\right)$

$\boxed{\tau = FL}$

* →

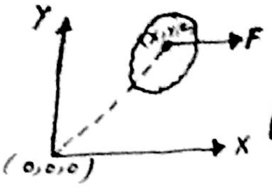


⇒ Trans (F) + Rotn (F2)

$\boxed{\tau = F_2 L}$

(iv) → If Force $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$ is applied on object at point x, y, z & It Rotate w.r.t (0,0,0)

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}}$$



$$\boxed{\begin{aligned} \vec{r} &= x\hat{i} + y\hat{j} + z\hat{k} \\ \vec{F} &= F_x\hat{i} + F_y\hat{j} + F_z\hat{k} \end{aligned}}$$

$$\boxed{\vec{\tau} = \hat{i}(yF_z - zF_y) - \hat{j}(xF_z - zF_x) + \hat{k}(xF_y - yF_x)}$$

$$\boxed{\vec{\tau} = \vec{\tau}_x\hat{i} + \vec{\tau}_y\hat{j} + \vec{\tau}_z\hat{k}}$$

$$\boxed{\begin{aligned} \tau_x &= yF_z - zF_y \\ \tau_y &= zF_x - xF_z \\ \tau_z &= xF_y - yF_x \end{aligned}}$$

(v) → Force applied at point (x_2, y_2, z_2) & It Rotate w.r.t point (x_1, y_1, z_1).

$$\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$\boxed{\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}}$$

$$\boxed{\vec{F} = F_x\hat{i} + F_z\hat{j} + F_z\hat{k}}$$

$$\boxed{\vec{\tau} = \vec{r} \times \vec{F}}$$

$$\boxed{\begin{aligned} \tau_x &= yF_z - zF_y \\ \tau_y &= zF_x - xF_z \\ \tau_z &= xF_y - yF_x \end{aligned}}$$

$$\boxed{\begin{aligned} x &= x_2 - x_1 \\ y &= y_2 - y_1 \\ z &= z_2 - z_1 \end{aligned}}$$

**

$$\tau = r\perp F = r\perp(ma)$$
$$= r\perp(m\alpha\, r\perp)$$
$$\boxed{\tau = I\alpha}$$
F M a (F = ma)

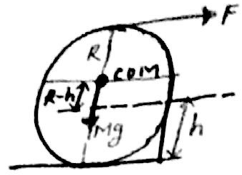
Sphere of Mass 'M' Radius 'R' is setup on stair of height 'h' than minimum value of tangential Force Applied on Sphere.

$$F(2R - h) \geq Mg(s)$$

$$\boxed{F \geq \frac{Mg(s)}{2R - h}}$$

**

$$\boxed{F_{min} = \frac{Mg(s)}{2R - h}}$$



AIIMS 2015, AIEEE

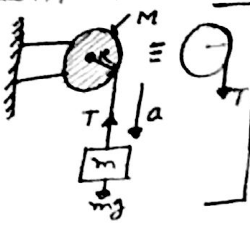
Tenson in string & Accln in this type system.

$$\boxed{a = \frac{2mg}{2m + M}} *$$

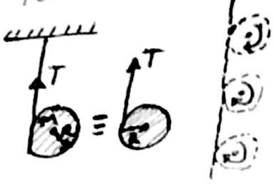
AIIMS *

$$\boxed{T = \frac{Ma}{2} = \frac{Mmg}{2m + M}}$$

$$\boxed{T = \frac{Mg}{\cos\theta} = \frac{Mg(1 + R)}{\sqrt{(1 - R)^2 - R^2}}}$$



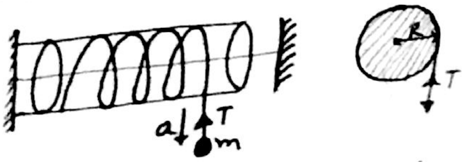
Tenson in a string & Accln of Disk.



$$\boxed{T = \frac{Mg}{3}} *$$

$$\boxed{a = \frac{2T}{M} = \frac{2g}{3}} *$$

Solid cylinder Rotate w.r.t axis Wich is based from COM & ⊥ to the length. When object of mass 'm' moving downward with the help of massles string. than Accleration of object & Tenson in string. (Mass = M, Radius = R).



$$\boxed{\tau = TR = I\alpha}$$

$$T_R = \frac{MR^2}{2}\left(\frac{a}{R}\right)$$

$$a = \frac{2T}{M} \quad -(1)$$

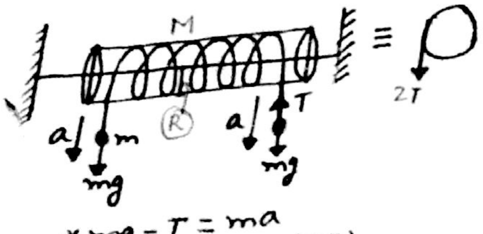
$$Mg - T = ma \quad -(1)$$

$$mg - T = m\left(\frac{2T}{m}\right)$$

$$* \boxed{T = \frac{Mmg}{M + 2m}}$$

$$* \boxed{a = \frac{2T}{M} = \frac{2mg}{M + 2m}}$$

Tenson in string & Accleration of Mass.



$$\tau = R \times F = R(2T)$$
$$= \left(\frac{MR^2}{2}\right)\left(\frac{a}{R}\right)$$

$$\boxed{a = \frac{4T}{M}}$$

$$\boxed{T = \frac{mMg}{M + 4m}} *$$

$$\boxed{a = \frac{4T}{M} = \frac{4mg}{M + 4m}} *$$

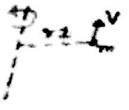
$$* \; mg - T = ma$$
$$mg - T = m\left(\frac{4T}{m}\right)$$

Angular Momentum (L) → Moment of linear momentum define angular momentum of object.

$$\boxed{\tau = \vec{r} \times \vec{p}} *$$

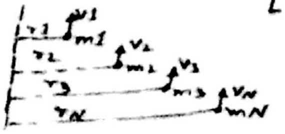
$$\boxed{L = I\omega} *$$

Angular Momentum of point Mass.



$$J = I\omega$$
$$= mr^2\left(\frac{v}{r}\right)$$
$$\boxed{J = mvr} *$$

Angular Momentum of point Mass system.



$$L = m_1v_1r_1 + m_2v_2r_2 + \cdots m_nv_nr_n$$

$$\omega(I_1 + I_2 + I_3 \cdots I_n)$$

$$\boxed{L = I\omega} *$$

Angular Momentum of Point (X, Y, Z) w.r.t (0,0,0)

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\vec{p} = P_x\hat{i} + P_y\hat{j} + P_z\hat{k}$

$\boxed{\vec{L} = I\omega}$ $\boxed{L = \vec{r} \times \vec{P}}$ $\Rightarrow L_x\hat{i} + L_y\hat{j} + L_z\hat{k}$

$$\boxed{\begin{aligned} L_x &= YP_z - ZP_y \\ L_y &= ZP_x - XP_z \\ L_z &= XP_y - YP_x \end{aligned}}$$

Linear motion	Angular motion
$\boxed{P = mv}$ $\boxed{W = \vec{F}\cdot d\vec{S}}$ $\boxed{\vec{P} = \vec{F}\cdot\vec{v}}$	$\boxed{L = I\omega}$ $\boxed{\omega = \vec{\tau}\cdot d\vec{\theta}}$ $\boxed{P = \vec{\tau}\cdot\vec{\omega}}$
$m = I$, $S = \theta$, $v = \omega$, $a = \alpha$	
* Translational K.E or Linear K.E: $\boxed{K.E = \frac{1}{2}mv^2 = \frac{P^2}{2m} = \frac{1}{2}Pv}$	* Rot^n K.E: $\boxed{K.E_R = \frac{1}{2}I\omega^2 = \frac{L^2}{2I} = \frac{1}{2}L\omega}$
* From Newton 2nd law: $\boxed{F_{ext} = \frac{d\vec{P}}{dt}}$	* From Newton 2nd Law: $\boxed{\tau_{ext} = \frac{d\vec{L}}{dt}}$
* M = const: $\boxed{F_{ext} = m\frac{d\vec{v}}{dt} = m\vec{a}}$	* I = const: $\boxed{\tau_{ext} = I\frac{d\vec{\omega}}{dt} = I\vec{\alpha}}$
* M = variable: $\boxed{F_{ext} = v\frac{dm}{dt}}$	* I = variable: $\boxed{\tau_{ext} = \omega\frac{dI}{dt}}$
* V, M = variable: $\boxed{F_{ext} = v\frac{dm}{dt} + ma}$	* I, ω = variable: $\boxed{\tau_{ext} = \omega\frac{dI}{dt} + I\alpha}$

Law of Angular Momentum conservation

$$\boxed{\tau_{ext} = \frac{d\vec{L}}{dt} = 0}$$

$dL = 0 \Rightarrow \boxed{L = const.}$

$\boxed{I\omega = const.}$

* $\boxed{I_1\omega_1 = I_2\omega_2}$

* $\boxed{\omega \propto \frac{1}{I}}$

**

$$\boxed{\begin{aligned} &* I\uparrow \Rightarrow \omega\downarrow \\ &* I\downarrow \Rightarrow \omega\downarrow \end{aligned}}$$

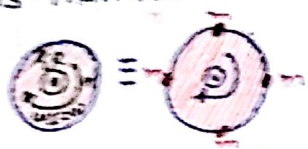
NOTE → In Q. Gentelly, suddenly, slowely, Instantly words are given then angular momentum conserved.

**

* $\boxed{\vec{F}_{ext} = 0}$

Disk of Mass 'M' Radius 'R' Rotate w.r.t Geometrical axis, suddenly Four point Mass 'm' placed on its circumference than new Angular velocity of Disc w.r.t same axis. ~~From value~~ If Initial value is 'ω'.

$\boxed{\tau_{ex} = 0} \Rightarrow \boxed{L_1 = L_2}$



$I_1\omega_1 = I_2\omega_2$

$\left(\frac{MR^2}{2}\right)\omega = \left[\frac{MR^2}{2} + 4(mR^2)\right]\omega_2$

* $\boxed{\omega_2 = \frac{M\omega}{M + 8m}}$

Dis of Mass 'M' Radius 'R' Fixed w.r.t its axel & person of Mass 'm' Ont at distance R/2 from its axel. If person move in a circular path of Radius R/2 with uniform speed 'V' than angular velocity of plateform.

$\boxed{L_1 = L_2}$

$0 = mv\left(\frac{R}{2}\right) + \left(\frac{MR^2}{2}\right)\omega_P$

* $\boxed{\omega_P = (-)\frac{mV}{MR}}$

opposite direction

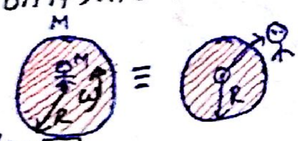
Person of Mass 'm' moves along the circumference of Marygoround (disk) of Mass 'M' Radius 'R' With uniform speed 'v' suddenly person Jump on disk than angular velocity of disk.

$\boxed{L_1 = L_2}$

$mvR = \left(\frac{MR^2}{2} + mR^2\right)\omega_2$

* $$\boxed{\omega_2 = \frac{mvR}{\frac{MR^2}{2} + MR^2}}$$

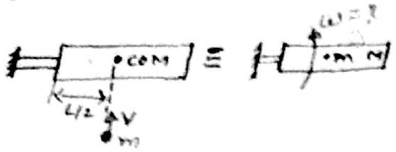
A Marry-go-Round, Made a Ring like plateform of Radius R & Mass 'M' is Revolving with angular speed 'ω'. A person of Mass 'M' is Standing on it, At one Instant the person Jump of the Round, Radially away From centre of Round. Then speed of Round Afterward is →



Ans → Person at centre so M.OI = 0

So After Jumping the person 'ω' Remain same i.e. = $\boxed{\omega}$

Rod of mass 'M' length 'L' placed on frictionless horizontal surface. A point mass 'm' moves with velocity 'v' strike at COM of rod & perform perfectelly inelastic callision. Than angular velocity of rod after callision. (If it is rotate w.r.t one end.)

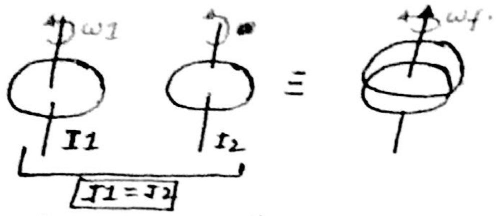


$\boxed{I_1 = I_2}$

$$mv\left(\frac{L}{2}\right) = I\omega$$
$$= \left[\frac{ML^2}{3} + m\left(\frac{L}{2}\right)^2\right]\omega_2$$
$$mv\left(\frac{L}{2}\right) = \left(\frac{ML^2}{3} + \frac{mL^2}{4}\right)\omega_2$$

$$\boxed{\omega_2 = \frac{6mv}{(4M+3m)L}} \ *$$

Disk of MOI 'I_1' rotate w.r.t geometrical axis with angular velocity ω_1. Another disk having MOI 'I_2' same axis placed on 1st disk coaxially. Than angular velocity of combination & loss in K.E.



$\boxed{I_1 = I_2}$

(i) → $\boxed{\omega_f = \dfrac{I_1\omega_1}{I_1+I_2}}$ *

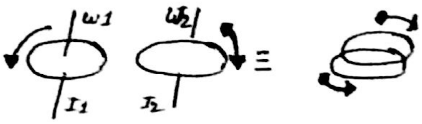
(ii) → $\boxed{\Delta K.E_{loss} = \dfrac{1}{2}\left(\dfrac{I_1 I_2}{I_1+I_2}\right)\omega_1^2}$ *

* Standard

$$\boxed{K.E_{trans} = \frac{1}{2}mv^2 = \frac{P^2}{2m} = \frac{1}{2}Pv}$$
$$\boxed{K.E_{rotn} = \frac{1}{2}I\omega = \frac{L^2}{2I} = \frac{1}{2}L\omega}$$

$$\boxed{\Delta K.E_{loss} = \frac{1}{2}\left(\frac{I_1 I_2}{I_1+I_2}\right)\left(\vec{\omega_1} - \vec{\omega_2}\right)^2}$$

MOI of disk is 'I_1' & rotate A.C.W with angular velocity ω_1 another disk of MOI 'I_2' angular velocity ω_2, C.W. gentelly placed on 1st disk than final angular velocity and loss in K.E.



$\boxed{I_1 = I_2}$ *

(i) → $\boxed{\omega_f = \dfrac{I_1\omega_1 - I_2\omega_2}{I_1+I_2}}$

(ii) → $\boxed{\Delta K.E_{loss} = \dfrac{1}{2}\left(\dfrac{I_1 I_2}{I_1+I_2}\right)(\omega_1+\omega_2)^2}$ *

Rotational Kinetic Energy

$$\boxed{K.E_{Rotn} = \frac{1}{2}I\omega^2 = \frac{L^2}{2m} = \frac{1}{2}L\omega}$$

Work Energy Theorum

$$\boxed{W_{N.C} + W_{ex} = \Delta K.E + \Delta U}$$

(a) → $W_{N.C} = 0,\ \Delta U = 0$

$$W_{ex} = \Delta K.E_r$$
$$\int \tau \cdot d\theta = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$$

(b) → $W_{ex} = 0 = W_{N.C}$

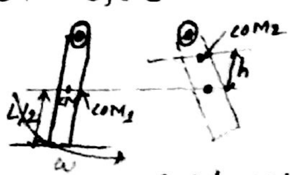
$$\boxed{0 = \Delta K.E + \Delta U}$$
$$\boxed{K.E_i + U_i = K.E_f + U_f}$$

Graph

(i) → $K.E_r$ v/s ω (I = const)
(ii) → $K.E_r$ v/s ω (L = const)
(iii) → $K.E_r$ v/s L (I = const)
(iv) → $K.E_r$ v/s L (ω = const)
(v) → $\log K.E_r$ v/s $\log \omega$ (I = const)

Imp → Work done to change angular velo. of disk 'ω' to '2ω' (mass 'M' radius 'R') rotate about geometrical axis → $\boxed{W_{ex} = \frac{3}{4}m\omega^2R^2}$ *

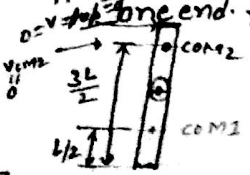
Rod of mass 'M' length 'L' hanged vertically from its one end. If angular velocity 'ω' is transfered at bottom point. than rise in height of its C.O.M.



$\boxed{W_{ex} = 0 = W_{N.C}}$

$$\boxed{h = \frac{\omega^2 L^2}{6g}} \ *$$

Rod of mass 'M' length 'L' hanged vertically w.r.t horizontal axis wich is pased from one end. then minimum velocity at bottom point, It will complete vertical circular path.



$$\boxed{\omega_b = \sqrt{\frac{6g}{L}}} \ *$$

$$\left[V_{CM} = \omega\left(\frac{L}{2}\right) = \sqrt{\frac{3}{2}gL}\right.$$

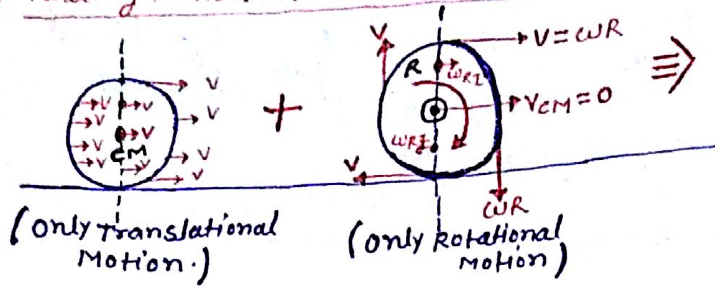
* velocity of rod at bottom point

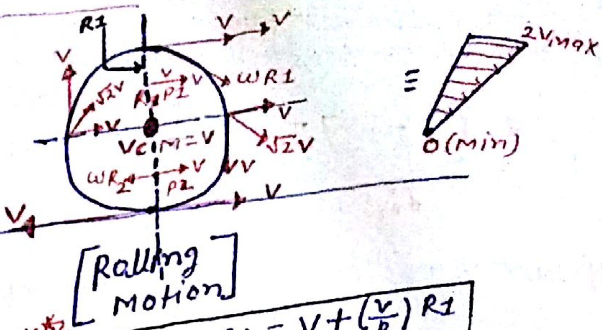
$\boxed{u = \omega L}$ *

$\boxed{u = \sqrt{6gL}}$

Rolling Motion [Translational + Rotational]

Rolling Motion on horizontal plane



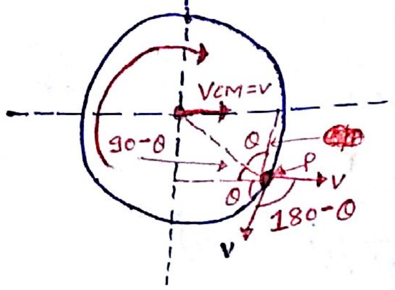


$$\boxed{\begin{aligned} V_{P1} &= V + \omega R_1 = V + \left(\frac{V}{R}\right) R_1 \\ V_{P2} &= V - \omega R_2 = V - \left(\frac{V}{R}\right) R_2 \end{aligned}}$$

NOTE → * When body Roll on horizontal plane then velocity of contact point Remain is always zero.

* Work done by Frictional Force at contact point is zero.

velocity of point 'P' at 'θ' angular position from vertical.



$$(V_R)_P = \sqrt{V^2 + V^2 + 2(V)(V)\cos(180 - \theta)}$$

$$= \sqrt{2V^2(1 - \cos\theta)}$$

$$= \sqrt{2V^2(2\sin\theta/2)}$$

$$\boxed{V_P = 2V\sin^2(\theta/2)}$$

Total Energy In a Rolling Motion

$$\boxed{T \cdot E = K \cdot E_T + K \cdot E_R}$$

$$K \cdot E_T = \frac{1}{2} m v^2$$

$$K \cdot E_R = \frac{1}{2} I \omega^2 = \frac{1}{2}(mK^2)\left(\frac{V}{R}\right)^2$$

$$K \cdot E_{Total} = \frac{1}{2} m v^2 + \frac{1}{2} m v^2 \left(\frac{K^2}{R}\right)$$

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$$\boxed{K \cdot E_{Total} = \frac{1}{2} m v^2 \left(1 + \frac{K^2}{R^2}\right)}$$

Part/Fraction of Total Energy

$$\frac{K \cdot E_T}{K \cdot E_{Total}} = \frac{\frac{1}{2} m v^2}{\frac{1}{2} m v^2 \left(1 + \frac{K^2}{R^2}\right)} = \frac{1}{1 + \frac{K^2}{R^2}}$$

$$\frac{K \cdot E_R}{K \cdot E_{Total}} = \frac{\frac{1}{2} m v^2 \left(\frac{K^2}{R^2}\right)}{\frac{1}{2} m v^2 \left(1 + \frac{R^2}{K^2}\right)} = \frac{1}{1 + \frac{R^2}{K^2}}$$

NOTE → * part or, Fraction of Translational or, Rotational Energy is Independent from size & mass of object.

* It only depend on its shape.

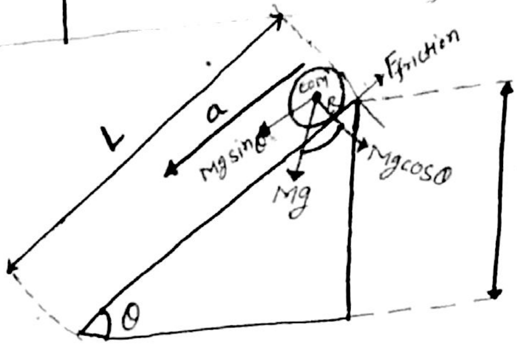
In a pure Rolling motion on horizontal plane Find out How much part of total Energy in Form of Translational & Rotational K.E. If object Ring, Disk, solid sphere, Hollow sphere, solid cylinder, Hollow cylinder.

$$\boxed{\underset{\text{Max / Min}}{\frac{K \cdot E_T}{K \cdot E_T}} = \frac{1}{1 + \underset{\text{Min / Max}}{\frac{K^2}{R^2}}}}$$

$$\boxed{\frac{K \cdot E_R}{K \cdot E_T} = \frac{1}{1 + \frac{K^2}{R^2}}}$$

Object	K^2/R^2	$\frac{\quad}{K.E_T} = \frac{\quad}{1+\frac{K^2}{R^2}}$	$\frac{\quad}{K.E_T} = \frac{\quad}{1+\frac{R^2}{K^2}}$
Ring or, Hollow Cylinder	$I = MR^2 = KR^2$ $\frac{K^2}{R^2} = 1$	$\frac{1}{1+1} = \frac{1}{2} = 50\%$	$\frac{1}{1+1} = \frac{1}{2} = 50\%$ ✓
DISK or, Solid Cylinder	$I = MR^2 = MK^2$ $\frac{K^2}{R^2} = \frac{1}{2} = 0.5$	$\frac{1}{1+\frac{1}{2}} = \frac{2}{3} = 66.66\%$	$\frac{1}{1+2} = \frac{1}{3} = 33.33\%$ ✓
Solid Sphere	$I = \frac{2}{5}MR^2 = MK^2$ $\frac{K^2}{R^2} = \frac{2}{5} = 0.4$	$\frac{1}{1+\frac{2}{5}} = \frac{5}{7} = 70\%$	$\frac{1}{1+\frac{5}{2}} = \frac{2}{7} = 30\%$ ✓
Hollow Sphere	$I = \frac{2}{3}MR^2 = MK^2$ $\frac{K^2}{R^2} = \frac{2}{3} = 0.66$	$\frac{1}{1+\frac{2}{3}} = \frac{3}{5} = 60\%$	$\frac{1}{1+\frac{3}{2}} = \frac{2}{5} = 40\%$ ✓

Rolling Motion on Incline plane



|i| → Accleration of particle

$$\boxed{a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}} \ *$$

|ii| → Time taken by particle to Reached at bottom

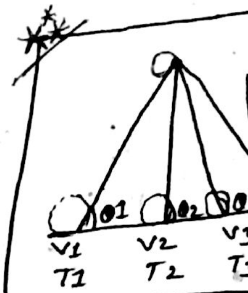
$$** \ \boxed{T = \sqrt{\frac{2L}{g\sin\theta}\left(1+\frac{K^2}{R^2}\right)} = \frac{1}{\sin\theta}\sqrt{\frac{2h}{g}\left(1+\frac{K^2}{R^2}\right)}}$$

|iii| → velocity at bottom

$$\boxed{V = \sqrt{\frac{2g(L\sin\theta)}{1+K^2/R^2}} = \sqrt{\frac{2gh}{1+K^2/R^2}}} \ *$$

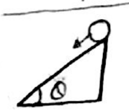
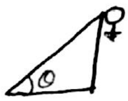
* It depend on Angle of Inclination.

***



$V_1 = V_2 = V_3$
$T_1 > T_2 > T_3$
$a_1 < a_2 < a_3$

***

Rolling Motion	Only Translational Motion	Falling
		
$a_1 = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$	$a_2 = g\sin\theta$	$a_3 = g$
$V_1 = \sqrt{\frac{2gh}{1+K^2/R^2}}$	$V_2 = \sqrt{2gh}$	$V_3 = \sqrt{2gh}$
$T_1 = \frac{1}{\sin\theta}\sqrt{\frac{2H}{g}\left(1+\frac{K^2}{R^2}\right)}$	$T_2 = \frac{1}{\sin\theta}\sqrt{\frac{2H}{g}}$	$T_3 = \sqrt{\frac{2H}{g}}$

**

$a_1 < a_2 < a_3$
$V_1 < V_2 = V_3$
$T_1 > T_2 > T_3$

Solid sphere, Hollow Sphere, Disk & Ring is drop from same height along the surface of Incline plane of same angle of Inclination then→
(i)→ Acc^n bottom (ii)→ velocity at bottom (iii)→ Time taken by particle to Reached at bottom.
(If particle perform pure Rolling on Incline plane).

$$* \frac{K^2}{R^2} \Rightarrow \boxed{\text{Ring} > \text{H. sphere} > \text{Disk} > \text{Solid sphere}}$$

**

$a_{Ring} < a_{H.S} < a_{Disk} < a_{S.sphere}$

$V_{Ring} < V_{H.S} < V_{Disk} < V_{S.sphere}$

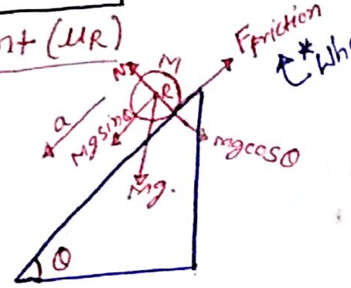
$T_{Ring} > T_{H.S} > T_{Disk} > T_{S.sphere}$

Rolling Friction / Friction coefficient (μ_R)

*

$$\boxed{F_{friction} = \frac{Mg\sin\theta}{1 + \frac{R^2}{K^2}}}$$

Rolling Friction Force



*Whatever its direction of motion its always Act upward.

NOTE→ Rolling Friction Force & Friction coefficient depend on shape of object.

Rolling Friction coefficient (μ_R)

*

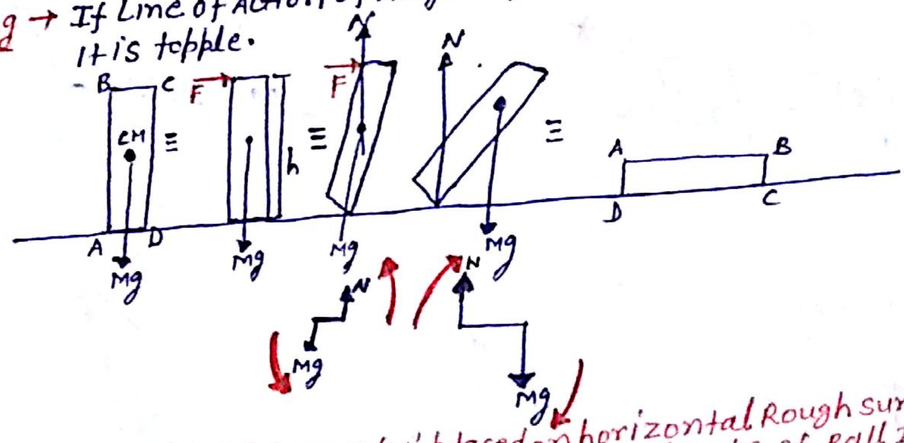
$$\boxed{\mu_R = \frac{\tan\theta}{1 + \frac{R^2}{K^2}}}$$

**

$$\boxed{\mu_R \rightarrow \text{Ring} > \text{H.sphere} > \text{Disk} > \text{S.sphere}}$$

AIEEE 2016

Toppling → If Line of Action of Weight of body coming out from its base then it is topple.



Exemplar based

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Billiard ball of Radius 'R' Mass 'M' placed on horizontal Rough surface. Force 'F' is applied 'h' height above from COM. Then Initial Acc^n of Ball If Magnitude of applied Force is 'F'

$F + f = Ma_i$ —(I)

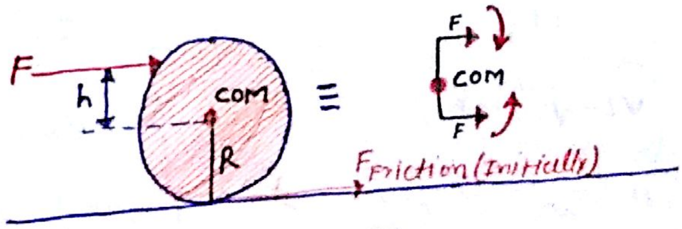
$F(h) - f(R) = I\alpha$

$Fh - f(R) = \left(\frac{2}{5}MR^2\right)\frac{a}{R}$

$Fh - f(R) = \left(\frac{2}{5}mR^2\right)a$ —(II)

$FR + fR = MRa$ —(I)×R

$F(h+R) = \frac{7}{5}MRa$



*

$$\boxed{a = \frac{5F(h+R)}{7MR}}$$

**

$$\boxed{a = \frac{5F}{7M}\left(1 + \frac{h}{R}\right)}$$

NCERT

Three body a Ring, a solid cylinder & solid sphere Roll down the same Inclined plane without slipping. The radi of these body are Identical. Wich of ~~these~~ body Reaches the ground with max. velocity.

$* \; V^2 = \left(\frac{2gh}{1+K^2/R^2}\right)$

* Independent from mass of Rolling body.

* For Ring $[K^2 = R^2]$, $V_{Ring} = \sqrt{\frac{2gh}{1+1}} = \boxed{\sqrt{gh}}$

* For solid cylinder $[K^2 = R^2/2]$

$* \; V_{disc} = \boxed{\sqrt{\frac{4gh}{3}}}$

~~solid cylin~~

* $K^2 = 2R^2/5$ (solid sphere)

*AIIMS

$V_{sphere} = \sqrt{\frac{2gh}{1+2/5}} = \boxed{\sqrt{\frac{10gh}{7}}}$

* velocity.

Ring < solid cylinder < s.sphere

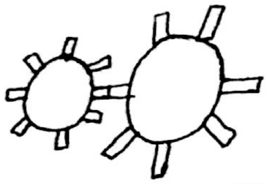
**

case-I

$\boxed{V1 = V2}$

$\boxed{\omega_1 R_1 = \omega_2 R_2}$

case-II



$\boxed{\omega = \text{same}}$

$\boxed{\frac{V1}{R1} = \frac{V2}{R1}}$